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BERTRAND PRICE COMPETITION IN A SOCIAL ENVIRONMENT***

BY

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Summary

We analyze the dynamic behaviour of firms that locally interact through price competition in a social environment in an evolutionary game-theoretic model. These firms update their prices according to the behavioural rule 'Win Cooperate, Lose Defect' (WCLD), which is often observed in experimental economics. It can be regarded as a generalized Tit-for-Tat strategy. The model can explain the simultaneous emergence of collusive behaviour, price dispersion and occasional local price wars. Price wars only last for a short period of time after which the firms start to collude again.

Key words: evolutionary game theory, local interaction, price competition, social interaction, tit-for-tat

1 INTRODUCTION

In many industries it is typical for firms (or shops) to compete directly with neighbouring firms and only indirectly with the other firms in the market. Literal examples are gasoline stations and bakery stores that compete locally for customers who live nearby but do not compete for customers that live far away. Or, in terms of horizontal competition, Bordeaux vineyards compete among each other for consumers who prefer Bordeaux wines but they compete less with vineyards from the Rhone valley or the Elzas. So, wine producers compete for niches with a certain taste characteristic and their closest competitors are those that offer an almost identical flavour. Heineken, Amstel, Oranje Boom, and Grolsch compete with different flavours of beer on the same Dutch beer market. Another example is telephone and internet companies which offer a wide range of tariffs, connections, and service menus to their costumers. In terms of horizontal competition 'neighbouring' means competing products that are relatively close in some abstract product space. As a consequence, the own price and the neighbouring prices

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matter in attracting customers. This paper focuses on competition between products with horizontal characteristics in a local interaction model.

For explanatory reasons we choose competition among Dutch gasoline stations as the leading example of local interaction. The Dutch gasoline market does not exhibit a uniform market price for gasoline, but instead there is price dispersion. It is well known that the price near the Belgium and German borders is lowered in order to avoid a price difference that would drive the Dutch gasoline stations close to the border out of business. Less well known is that relatively far away from the German and Belgium borders, prices may also differ locally. Furthermore, in some regions local price wars occur while at the same time collusive behaviour among stations is observed in other regions. Standard economic theory can explain collusion among all stations by a repeated game argument and price dispersion by either introducing search costs among costumers or heterogeneity among local population densities, see e.g. Tirole (1988). In order to explain the simultaneous occurrence of collusive behaviour and price wars we will apply an evolutionary game-theoretic model, see e.g. Samuelson (1997), Vega-Redondo (1996), Young (1998), or the survey article by van der Laan and Tieman (1998). For evolution in *local interaction models* see e.g. Ellison (1993, 2000) or Eshel, Sansone, and Shaked (1999).

In this paper we study local interaction of firms over time in which firms compete through prices and produce heterogeneous goods that are close substitutes. Prices are assumed to be discrete and the location of each firm remains fixed over time. The firms are located on a torus, which is roughly speaking a two-dimensional version of a circle with firms located equidistantly. All firms are thus in symmetric situations, except for their location. Firms only compete with firms in a subgroup of the population, called their *neighbours*. A firm's neighbours are all firms located in positions on the torus directly adjacent to the location of the firm. The group of neighbours is different for each firm, although there may be substantial overlaps between the groups of neighbours of different firms. Therefore, each firm is indirectly linked with any other firm through a chain of firms that are each others' neighbours.[1] Although we specify behaviour on the microlevel we study its consequences at the macrolevel. So, a link between the behaviour of individual firms and the behaviour of the population as a whole is established. This approach is known in social psychology as the *level of analysis approach* (see e.g. Messick and Liebrand (1995)), and it has been addressed in economics by e.g. Schelling (1978), Samuelson (1997), and Young (1998).

Standard economic theory assumes profit maximization and (hyper)rationality of the firms, i.e. all firms are able to foresee or predict (most of) the consequences of current behaviour on future payoffs and they are able to calculate the correct equilibrium outcome given their predictive powers. In this paper we drop

1 This can be interpreted as firms levying externalities on non-neighbouring other firms, see e.g. Ginsburgh and Keyzer (1997) for a treatment of externalities in a general equilibrium framework.

profit maximizing behaviour and (hyper)rationality. Instead, we propose that the firms follow a simple behavioural rule (or heuristic) in updating their prices, as is often done in the field of behavioural game theory (see e.g. Camerer (1997)). The novelty in this paper is that a simple behavioural rule is assumed that originates from experiments, both in economics (see e.g. Offerman, Sonnemans, and Schram (1996)) and in social psychology (see e.g. Messick and Liebrand (1995)). This update rule is known as *'Win Cooperate, Lose Defect'* (WCLD). In every round of our model, one of the firms is given the opportunity to adjust its current behaviour. We refer to this firm as the *subject*. Based upon the subject's profits resulting from competing with its *cheapest* neighbour it updates its price. Under WCLD, that firm compares its last profit with the average profit of its neighbours the last time they competed as the subject. If the subject's profit lies above this average, the firm is said to be in a '*win*' situation, it infers that its neighbours competed in a collusive manner and the firm responds by behaving more collusive in the near future by adjusting its price upwards. If the subject's profit falls short of this average, it is in a '*lose*' situation, the firm will deduct that its neighbours behave more competitively and it responds by setting a more competitive price itself, i.e., by lowering its price. Thus, WCLD inhibits reciprocal behaviour, because competitive behaviour of the subjects neighbours is implicitly punished by competitive behaviour of the subject. In fact, WCLD can be regarded as a multi-agent generalization of the well know *Tit-for-Tat* (TfT) update rule in the prisoners' dilemma, which is also reciprocal (see e.g. Axelrod (1987)).

The WCLD update rule is a rule of procedural rationality[2] as described by Simon (1976), and it is therefore a behavioural rule according to Camerer (1997), who describes such a rule as a description of actual behaviour that is driven by empirical observations (mostly experiments), and charts a middle course between over-rational equilibrium analyses and under-rational adaptive analyses. An alternative rationale for WCLD can be found in the literature on aspiration levels (see e.g. Karandikar, Mookherjee, Ray, and Vega-Redondo (1998), Palomino and Vega-Redondo (1999), Rabin (1993), Van Lange, De Bruin, Otten, and Joireman (1997), Van Lange (1997), Thibaut and Kelley (1959), and Kelley and Thibaut (1978)). An *aspiration level*, translated from game theory to price competition, is the minimum profit a firm requires in order to set a certain price. How the firm updates its price depends upon whether or not its profit falls short of its aspiration level. The update rule WCLD results if the aspiration level is set equal to the average profit the subject's neighbours got the last time they played the game as subject. The aspiration level under WCLD is thus endogenous. Whenever the subject gets a higher profit than the reference profit of its set of neighbours, the firm will tend

2 Procedural rationality means that people think about their choices before they come to a decision. It focusses on the process or path leading to an outcome. Substantive rationality focusses on the outcome. It supposes that people act as utility maximizers. The common approach in economics is substantive rationality.

to collusion. When its profit falls short of this number, the firm will play more competitively in the next round of play.

Our model yields a high-dimensional martingale process which has no analytical solutions in closed form. Therefore, we study the macroeconomic consequences of individual behaviour at the microlevel through performing simulations for different parameter values. We define the notion of a *stable state* at the aggregate level as a situation in which the percentages of firms which set a certain price is (almost) constant over time. Interest goes out to the stable state of the population and the path towards this stable state. Our simulations show that a stable state always emerges, that it is unique, displays a large percentage of collusive behaviour and such a state always corresponds to *price dispersion*. So, even though all firms are symmetric, non-uniform prices arise. Since the notion of stability is defined on the aggregate level it does not mean that all firms stick to the price they have set forever after. Instead even in a stable state individual firms still change their price.[3] The population is in perpetual flux and individual changes more or less cancel each other out on the aggregate level. The simulations also show that occasionally a *price war* occurs, during which the population is away from the stable state. In fact, only a few local changes can cause a local price war, which locally upsets the collusive behaviour. This war may spread out rapidly over a large part of the population. Still, price wars in one region and collusive behaviour somewhere else coexists. In the medium run the population will again evolve towards the same stable state it was in before the price war started. The results are very general, in the sense that varying the parameter values does not alter the qualitative results. So, our simulation results give an explanation for the stylized facts observed in the Dutch gasoline market.

This paper is organized as follows. Section 2 introduces the price competition model and discusses its underlying assumptions. Section 3.1 reports the simulation results in case each firm can only choose between the competitive price and the collusive cartel price. This case is strategically equivalent to a prisoners' dilemma. In section 3.2 the focus is on the simulation results for the general case with more than two prices. This section also elaborates on the stability of the results. Section 4 shows how a price war might start in a stable state. Finally, section 5 concludes.

2 THE MODEL

Our model is best understood if we first consider n firms located on the surface of a circle. A firm's location specifies this firm's product characteristics in terms of horizontal differentiation. We assume that the firms are evenly located over the surface. Adjacent firms are *neighbours* and, thus, each firm has two neighbours.

3 Similar features emerge in models with stochastic difference equations with thresholds, see e.g. Ermoliev, Keyzer, and Norkin (2000) and the references therein.

The neighbours produce competing products that are relatively close substitutes. The surface of the circle is one-dimensional and can be represented by the interval $[1,n]$, called a one-dimensional *torus*, as follows. Firm 1 is located at $x = 1$, firm 2 at $x = 2$, etc. Thus, firm i, $i = 2, \ldots, n-1$, at location $x = i$ has firms $i-1$ and $i+1$ as its neighbours. Firm 1 has firm 2 and n as its neighbours and, therefore, firm 1 and $n-1$ are the neighbours of firm n, thereby connecting the two endpoints of the interval. Instead of considering a circle (or one-dimensional torus) we investigate horizontal differentiation modelled as firms located symmetrically on a two-dimensional surface. We can either choose the surface of a sphere or a two dimensional torus. The latter is mathematically more convenient, because it can be represented as the integer grid on the rectangular $[1,n] \times [1,n]$ for some integer $n \geq 3$. See Figure 1 for an illustration. We assume that each firms's location on the two-dimensional torus is given uniquely by a pair $x = (x_1, x_2)$ with $x_1, x_2 \in \{1,\ldots,n\}$. There is a degree of freedom in defining a firm's neighbours. We consider as a firm's neighbours the eight adjacent firms in the north, northeast, east, southeast, south, southwest, west and northwest direction. For example the firm located at (2,2) has (1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), and (3,3) as its neighbours. Similar as in the one-dimensional case, firms with the first coordinate equal to 1 (i.e. $x_1 = 1$) have neighbours with first coordinate equal to n (i.e. $x_1 = n$) and *vice versa*. Also, firms with $x_2 = 1$ have neighbours with $x_2 = n$ and *vice versa*. Thus, the eight neighbours of the firm located at (1,1) are given by (n,n), $(n,1)$, $(n,2)$, $(1,n)$, (1,2), $(2,n)$, (2,1), and (2,2). Formally, the eight neighbours of the firm located at $x = (x_1, x_2)$ are the firms on the locations $y = (y_1, y_2)$, except $y = x$, given by

$$\{y\colon y_1 = (x_1 - 1) \bmod n, \quad x_l, \quad (x_l + 1) \bmod n, \quad l = 1,2\}.$$

We imagine an economy in which each location also represents a small community (all communities being equal) with one gasoline station. The customers of community x either visit the gasoline station at x or go to the *cheapest* neighbouring gasoline station (and are assumed not to travel further). The duopoly is specified as follows. We assume $\gamma_0 > 0$, $\gamma_1 > \gamma_2 > 0$ and a linear demand function $D_i(p_i, p_j) = \gamma_0 - \gamma_1 p_i + \gamma_2 p_j$ with p_i the price of firm $i = 1,2$ and p_j the price of its cheapest opponent j, $j = 1,2$, $j \neq i$. A linear demand function is consistent with price competition in a horizontally differentiated market with two firms and quadratic transportation costs, see e.g. Tirole (1988). So, firm i's demand is declining in i's own price, rising in its competitors j's price and it is more sensitive to firm i's own price than it is to the price of its opponent. Furthermore, each firm's cost function is linear and given by $C_i(q_i) = q_i$, $i = 1,2$, where q_i denotes firm i's production. The profit of firm i is equal to

$$\begin{aligned}\pi^i(p_i, p_j) &= D_i(p_i, p_j) \cdot p_i - C(D_i(p_i, p_j)) \\ &= -\gamma_1 (p_i)^2 + (\gamma_0 + \gamma_1) p_i + \gamma_2 p_i p_j - \gamma_2 p_j - \gamma_0,\end{aligned}$$

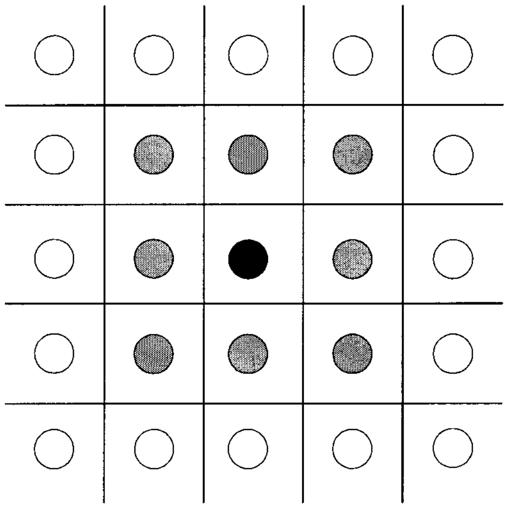

Figure 1 – An illustration of a part of the torus. Every circle represents a producer. The neighbours of the producer indicated by the black circle are shaded.

which is quadratic and concave in p_i and linear in p_j. This duopoly admits a unique and symmetric Nash equilibrium. Denote each firm's Nash equilibrium price as p^N and let $p^C > p^N$ be the cartel price which is set by both firms when maximizing their joint profits. In most of the simulations we have set the demand parameters to be $\gamma_0 = 20$, $\gamma_1 = 1$ and $\gamma_2 = \frac{1}{2}$, leading to $p^N = 14$ and $p^C = 20\frac{1}{2}$.

We assume that the set of prices is discrete. First, we restrict attention to prices in the interval $[p^N, p^C]$ only. The reason is that whenever both firms set the same price $p_i = p_j = p$, then the profit of firm i is decreasing in his own price p_i for p above p^N and increasing in p_i for p below p^N. Hence, a price below p^N seems unreasonable. Furthermore, the joint profit is decreasing in p when $p > p^C$ and increasing in p when $p < p^C$. So, collusion on $p > p^C$ seems unreasonable as well. Next, we choose $\frac{1}{k}(p^C - p^N)$. For some integer $k \geq 1$, as the step-size of the grid on the interval $[p^N, p^C]$. The prices are labelled from 0 to k leading to a discrete set of $k+1$ prices. The a-th price, $a = 0, \ldots, k$, of firm i corresponds to $\frac{k-a}{k} p^N + \frac{a}{k} p^C$ and will be referred to as price a. Thus, $a = 0$ and $a = k$ yield the Nash equilibrium price p^N and the cartel price p^C. We regard a as a measure for collusive behaviour and we will say that price $a+1$ is more collusive than price a.

The discrete set of prices implies that the two firms play a symmetric $(k+1) \times (k+1)$ bimatrix game in which the payoffs are determined by their Bertrand duopoly profits. For $k = 1$ this bimatrix game reduces to a standard prisoners' dilemma (PD) in which the competitive price p^N and the cartel price p^C can be identified as non-cooperative and cooperative behaviour respectively and p^N strictly dominates p^C for each firm. Table 1 illustrates the profits in a PD for the row-player for the parameters $\gamma_0 = 20$, $\gamma_1 = 1$ and $\gamma_2 = \frac{1}{2}$. Note that for $k > 1$ the bimatrix game also possesses what we call a 'prisoners' dilemma structure': The pair of prices (0,0) form the unique Nash equilibrium, price a, $a = 0, \ldots, k-1$, strictly dominates price $a+1$ and the pair of prices (k, k) maximizes the joint profit, i.e. it is the unique symmetric Pareto-efficient outcome (from the point of view of the two firms).

TABLE 1 – THE ROW-PLAYER'S TABLE OF PROFITS FOR THE CASE $k = 1$.

Subject\ Competitor	p^C	p^N
p^C	190.1	126.8
p^N	211.3	169.0

We now introduce the dynamics of the model. Time is discrete and labelled $t = 0, 1, 2, \ldots$ In each period one of the firms is selected randomly with each individual firm being equally likely to be selected. The selected firm at time t is called the *subject* at time t. The subject at time t competes with its cheapest neighbour. This is modeled as the Bertrand duopoly in *discrete* prices introduced above.

At each round the price of each firm is given and determined by history. For our purposes, we summarize the history at round t by the state s^t, which is defined as the pair of $n \times n$ matrices $(A(t), B(t))$, where the (i,j)-th element of the matrix $A(t)$ and $B(t)$ denotes the current price set by the firm at location (i,j) at time t, and the last profit realized by the firm at this location being the subject. Thus, the state space is

$$S = \{(A, B) \big| A \in \{0, 1, \ldots, k\}^n \times \{0, 1, \ldots, k\}^n \quad \text{and} \quad B \in \mathbb{R}_+^n \times \mathbb{R}_+^n\}$$

and $a_{i,j}(t)$ and $b_{i,j}(t)$, respectively denote the (i,j)-th element of $A(t)$ and $B(t)$. The population of firms starts in a given state $(A(0), B(0)) \in S$ at $t = 0$, which is chosen randomly in the simulations.

At each time t the subject gets the possibility to update its price, a so called *learning draw*. The subject competes with its neighbour who has set the *lowest* price. After the price competition at time t the subject compares its profit, labelled π_{self}, with the average profit of all of its neighbours realized the last time they played the game as the subject, labelled π_{nbs}. For ease of discussion, in the following we say that a firm is in a 'win' ('lose') situation, whenever its own profit π_{self} is higher (lower) than the reference profit π_{nbs} of its neighbours. This does not mean that firms that 'win' are (in the long run) better off than firms that 'lose'. Based on the comparison of the profits, the subject updates its price using the update rule 'Win Cooperate, Lose Defect'. Whenever the subject, say firm (i,j), is in a 'win' situation it sets its price $a_{i,j}(t+1) = a_{i,j}(t) + 1$ if $a_{i,j}(t) < k$ and sticks to its current price $a_{i,j}(t+1) = a_{i,j}(t)$ if $a_{i,j}(t) = k$. When $\pi_{self} < \pi_{nbs}$, the subject is in a 'lose' situation and updates its price $a_{i,j}(t+1) = a_{i,j}(t) - 1$ if $a_{i,j}(t) > 0$ and to $a_{i,j}(t+1) = 0$ if $a_{i,j}(t) = 0$. When both profits are exactly equal, the subject will stick to the price set at round t in the next round, i.e. $a_{i,j}(t+1) = a_{i,j}(t)$. Furthermore, $b_{i,j}(t+1) = \pi_{self}$. All the other firms do not update their prices nor their profits, i.e. their entries in the matrices A and B remain constant at round t.

Before analyzing this dynamic process, we first discuss the behavioural rule WCLD. The subject that is in a 'win' situation can easily infer that on average its neighbours behave more collusively. The rule WCLD implies that the subject responds by behaving more collusive in the near future. Similarly, in a 'lose' situation, the subject infers that its neighbours behave more competitively and the firm responds by setting a more competitive price itself. Hence, WCLD inhibits *reciprocal* behaviour, because competitive behaviour of the cheapest neighbour is implicitly punished by competitive behaviour of the subject. WCLD is also *forgiving*, because if the neighbours behave more collusively than the subject then the subject will be in a 'win' situation and will reward its neighbours by setting a more collusive price itself. The behavioural rule WCLD is a generalization of the well known *Tit-for-Tat* (TfT) update rule in the PD (see e.g. Axelrod (1987)), which is also reciprocal and forgiving. This can be easily seen as follows. Consider $k = 1$, i.e. a PD, and two firms located on a circle which play the PD of Table 1 as a repeated game according to WCLD and which *both* update their price after each round (which differs from our model). Suppose that the initial state consists of the price p^N and p^C for firm 1 and firm 2, respectively and the associated profits 211.3 and 126.8, respectively. Then, if both firms update simultaneously after each round, firm 1 (firm 2) is in a 'win' ('lose') situation after round 1 and both firms switch prices. By symmetry, they switch prices every round, which is exactly the behaviour prescribed by the TfT rule if we would start in the same initial state. However, in contrast to TfT, WCLD exhibits a random component, it allows only one firm to update per round and it also depends on the subject's profit, i.e. on the price the subject sets itself.

The dynamics specified above constitutes a Markov chain on the state space S, see e.g. Freidlin and Wentzell (1984) or Van Harn and Holewijn (1991). The number of states in this state space is, even for a small population, (for all practical purposes) much too large to calculate an invariant probability measure. Therefore, when analyzing the evolutionary process, we focus on an aggregate state space $\tilde{S} = [0,1]^{k+1}$. Element $\tilde{s}^t = (\tilde{s}^t_0, \tilde{s}^t_1, \ldots, \tilde{s}^t_k) \in \tilde{S}$ is a vector consisting of fractions $\tilde{s}^t_i$, $i = 0, 1, \ldots, k$, $t \in \mathbb{N}$, of the firms that play price i at time t. We define a stable state (relative to $\tilde{S}$) as follows.

Definition 1 *A population is in a stable state from time T on, when there exists a vector of population fractions* $\bar{s} = (\bar{s}_0, \bar{s}_1, \bar{s}_k)$, $\sum_{i=0}^{k} \bar{s}_i = 1$ *such that*

$$\left|\tilde{s}_i^{\,t} - \bar{s}_i\right| < \epsilon,\ \forall i,\ \forall t > T,$$

where $\epsilon > 0$ *is a fixed small simulation constant.*

Note that even when the population is stable, individual firms still get the opportunity to update their prices according to the update rule. Thus a stable state does not necessarily imply that all firms stick to their current prices forever. It simply means that after a specific time T, the population fractions do not vary more than a small amount.

A stable state is reached when for every possible price the number of firms abandoning this price to play another one (outflow) is equal to the number of firms switching to this particular price (inflow). This criterion is known as the *perpetual flux* (PF) criterion (see e.g. Palomino and Vega-Redondo (1999)) meaning that no fluctuation is observed at the aggregate level while individual firms may still change their prices at the microlevel. The PF criterion holds for all components of the stable state $\bar{s}$ and it is a reformulation of the stable state criterion of a Markov chain on the state space $\tilde{S}$. We use this criterion on the level of a *simulation run* to detect a stable state, where a simulation run is defined as the fixed number of rounds equal to some large constant, denoted as L, times the number of firms in the social environment (n^2). Therefore, on average each firm is selected to be the subject L times during one simulation run. An entire simulation consists of a large number of simulation runs. The total number of rounds in an entire simulation is equal to *number of simulation runs* $\cdot L \cdot n^2$. Since a constant *average percentage of collusion*, $100\% \cdot \sum_{i=0}^{k} [\tilde{s}_i^t(t) \cdot i]$, is a necessary condition for the PF criterion to hold, we implemented a check on the variation in the average percentage of collusion between one simulation run and the next simulation run, before the PF criterion is checked. The extra criterion is added for reasons of calculation, so the simulation program does not check for the PF criterion unless there is a possibility the criterion actually holds.

3 THE SIMULATION RESULTS

3.1 *Price Competition as a Prisoners' Dilemma*

In this section we report results for the case of price competition with two prices, i.e. $k = 1$, and profits as reported in Table 1. We simulated the PD for $n = 30$, i.e. 900 firms. For each set of parameter values, we performed 300 simulation runs, each having $L = 50$, and checked for stable states.

Table 2 – The simulation results for $k = 1$ and $n = 30$

Price	% of firms
p^N	49.8
p^C	50.2

In all of our simulations the state $\tilde{s}_i^t$ converges to the stable state summarized in Table 2. This stable state is characterized by a non-uniform price and collusive behaviour, where approximately half of the firms set the cartel price, while the other half set the competitive Nash equilibrium price.

Varying the demand parameters did not result in qualitatively different results. Raising the size of the torus n increases the convergence times, i.e. it takes more time to reach a stable state. We also investigated the effect of varying the size of the neighbourhood. We extended the number of neighbours for each firm to 24, by considering the firms that can be reached in two steps, i.e. we take the firm's direct neighbours together with the firm's neighbours' neighbours as the set of neighbours. We also considered the set of 48 neighbours, which consists of all the firms that can be reached in three steps. Entending the size of the neighbourhood results in a higher speed of convergence to the stable state, but does not change the stable state. The explanation of this result is postponed to the end of section 3.2.

3.2 *Competition with Multiple Prices*

In this section we focus on competition with more than two prices, i.e. $k \geq 2$. We simulate the model with the same parameter values as described in section 3.1. Except for variations in these parameter values, we focus attention on the influence of varying the value of the parameter k.

Surprisingly, in case $k \geq 6$, we observe behaviour close to a stable state for a relatively long period of time in which occasionally a price war starts, leading the state away from the stable state, but after a short time the state returns to the stable state just left. Therefore, we will refer to such states as quasi-stable states. We define a *quasi-stable* state as a state $\bar{s}$ that satisfies definition 1, except for recurring small periods of time. So, in the quasi-stable state $\bar{s}$ the firms behave most of the time according to a state $\tilde{s}^t$ very close to the state $\bar{s}$, but every once in a while there is a relatively short interval $[t_1, t_2]$ of time in which the state $\tilde{s}^t$ is further away from $\bar{s}$.[4] During the latter time interval there is an increase in competitive behaviour compared to the quasi stable state $\bar{s}$. Therefore, we call

4 Here, notions as 'small interval' and 'most of the time' should be taken loosely. Explicit upper and lower bounds for the time intervals $[t_1, t_2]$ have not been calculated.

such a time interval a *price war*, and say that the population is in the collusive phase when it is not involved in a price war. So, the quasi-stable state is equivalent to the collusive phase.

For values $1 \leq k \leq 5$ quasi-stable states and price wars do not occur and the firms' behaviour converges to a stable state. However, for $k \geq 6$, we report convergence to quasi-stable states and occasional price wars. For n large enough, the price wars appear only locally. For small tori ($n < 6$ approximately), these local effects can easily spread out over the whole population and thus cause global instability. To avoid these effects, we chose to simulate in much larger environments, mostly with $n = 30$.

We illustrate the outcome of the simulations by describing the case $k = 10$, that is the number of possible prices is 11 running from the competitive price $a = 0$ to the collusive price $a = 10$. In this case there is a very quick convergence towards a state in which most firms set the prices 8 and 9, while some firms set prices 7 or 10 and only a few firms set a price $a \leq 6$ but none of them sets the Nash equilibrium price $a = 0$. When this state is reached, convergence towards the ultimate quasi-stable state slows down. In the quasi-stable state firms set one of the prices 6, 7, 8, 9 or 10. For $n = 30$ the corresponding percentages of the population playing one of these prices is shown in Table 3. So, WCLD implies collusive behaviour with almost all mass on the top three prices. In small populations (e.g. $n = 10$) often only the prices 8 and 9 are observed in the quasi-stable state. The convergence to these quasi-stable states takes only a short time and it is reached within a few simulation runs.

Table 3 – The simulation results for $k = 10$ and $n = 30$

Price a	% of firms
6	0.4
7	8.7
8	28.2
9	38.5
10	24.2

After the quasi-stable state is reached the firms' behaviour stays near the state $\bar{s}$ for a stochastic period of time, after which there is a move away from state $\bar{s}$. The rare state of the population $\tilde{s}^t$ that causes this phenomenon is called a *trigger point*. In section 4 we further elaborate on the trigger point. When the state $\bar{s}$ is left, a sudden rapid increase in more competitive behaviour appears. After a short time however, the state of the population starts to converge towards the quasi-stable state $\bar{s}$ again. The behaviour of the firms immediately after the trigger event really is a *price war*. One firm feels forced to lower its price below the 'stable' level and becomes the cheapest neighbour for all of its neighbours. As a consequence more consumers will buy its product and its neighbours will be more frequently in a 'lose' situation when they compete with this firm. Then they also lower their price, thereby starting a downward spiral of the prices locally which may spread out to their neighbours, etc. After a price war has lasted for a stochastic time (longer in a small population, shorter in a large population) the affected firms all get a lower profit than they used to have and by playing against similar firms with low profits, they will more often realize a higher profit than the average profit their neighbours realized the last time they played the game as subject. Then firms will again begin to display more collusive behaviour. The convergence process towards the collusive phase of the quasi-stable state has started again. The state with low prices, which occurs at a price war, is not (quasi-)-stable.

Quasi-stable states occur for all values k from 6 up to 35.[5] In particular the simulations show that for increasing values of possible prices (up to $k \leq 35$), convergence towards the quasi-stable state still evolves essentially according to the same pattern observed for $k = 10$. First, there is convergence to a state in which most firms set one of the higher prices a (but this state is not the quasi-stable state) and after this initial, relatively fast convergence, convergence at a slower rate towards the ultimate quasi-stable state takes place. For all $k \leq 35$, in the collusive phase almost all firms set a high price a, corresponding to prices in the upper quarter of the interval $[p^N, p^C]$. A few typical frequency distributions of the prices in the quasi-stable state are reported in Appendix A. The average percentage of collusion in the (quasi-)stable state, rises monotonously with k, from 50% for $k = 1$ up to approximately 97% for $k = 35$, as can be seen in Figure 2. Since higher values of k can be interpreted as a weaker response of the firms to the 'win' and 'lose' situation, we conclude that a weaker response of a firm that updates leads to a higher ultimate degree of collusion.

Similar as reported in section 3.1 we performed comparative statics analysis by extending the neighbourhood size to 24 and 48 neighbours. In general, for most values of k the quasi-stable state that is reached with large neighbourhoods slightly differs from the one that is reached with eight neighbours resulting in somewhat less price dispersion. So, local interaction enhances price dispersion. This result is intuitively appealing, since a wider scope for the individual firm makes the persistence of small areas with different behaviour much more difficult and therefore should guarantee that there is less variation in prices across the population.

5 When the number of possible prices becomes larger than 36, the average percentage of cooperation in the population does not increase further. More strikingly, there may arise equilibria in which a lot of competitive behaviour is observed (defect equilibria). The sudden appearance of defect equilibria for values $k > 35$ is very surprising. The more because it only seems to be present in a setting where producers have only a small number of neighbours they can interact with. We will work on the explanation of this phenomenon in future research.

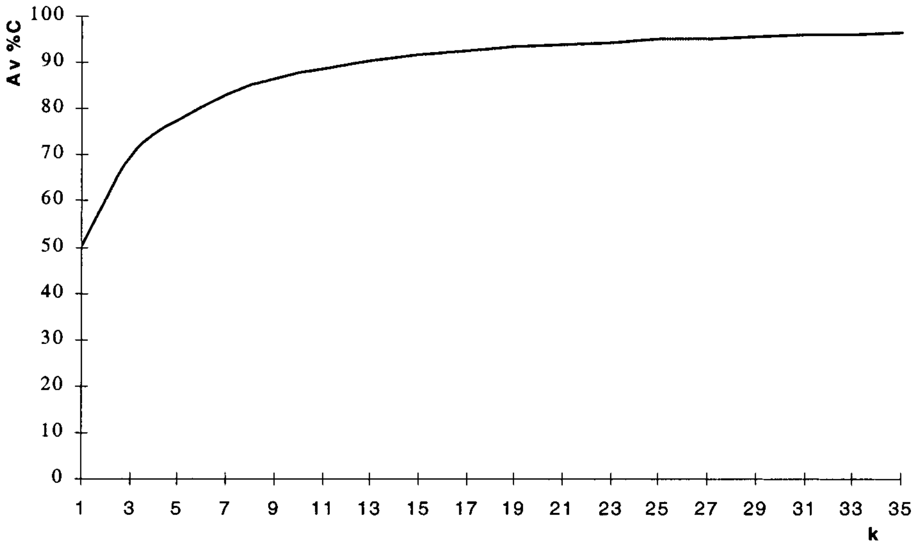

Figure 2 – The average percentage of collusion in the quasi-stable state for values of k from 1 to 35.

Similar as reported in section 3.1 we find that the speed of convergence towards the quasi-stable state rises as the size of the neighbourhood increases. This result is in contrast with Ellison's (1993, 2000) result on convergence rates in models with local interaction and Darwinian dynamics. Ellison concludes that convergence gets slower when the size of the neighbourhood increases. A plausible explanation is the following. The main effect in Ellison's model with Darwinian dynamics is that a larger neighbourhood size implies that it is harder for an exogenous mutant price to gain foothold in a population that is in equilibrium. This effect is of lesser importance in our model, since the essential mechanism behind our result has nothing to do with the presence of exogenous mutants. The effect described above, that a larger neighbourhood causes a tendency towards more homogeneous prices, seems to be more important in our model. This effect causes the initial convergence towards a state nearby the quasi-stable state to be swift, thus resulting in faster convergence altogether.

4 PRICE WARS EXPLAINED

As reported in section 3.2, a price war starts when the population in the quasi-stable state reaches a trigger point. In this section we identify such a trigger point, i.e. state, for $k = 10$ and explain how it evolves into a price war. We will illustrate this by means of five figures of the same 3×4 subsection of the population. All figures in this section represent the state space $s^t = (A(t), B(t))$ for the firms in the subsection, which are labeled (i,j), $i = 1,2,3$, $j = 1, \ldots, 4$ for convenience. Each box represents the price $a_{i,j}(t)$ firm (i,j) intends to set in the upper-left corner and the profit $b_{i,j}(t)$ in the bottom-right corner, which is the profit firm (i,j) realized the last time it competed as the subject.[6]

9 189	9 189	9 189	9 189
9 189	9 189	8 184	9 189
9 189	9 189	9 189	9 189

Figure 3 – The 3×4 subsection: The initial situation

Figure 3 represents the initial situation just before the trigger point might be reached. All firms set either price 8 or 9, which is in accordance with the stable state reported in Table 3. The subject's profit of the interaction between the subject (S) and the cheapest competitor (C), when they are both playing one of the prices 8 or 9 is given by[7]

S\C	8	9
8	189.28	195.20
9	183.79	189.91

Furthermore, the initial situation of Figure 3 has 11 out of the 12 firms with 189.28 as the last profit, which can arise after they all set price 8 the last time as the subject and all have encountered a cheapest neighbour that also set price 8. The firms that were in a 'win' situation changed their price into 9. The firm at position (3,2) has set price 9 against price 8 of the cheapest opponent and hence realized a profit of $\pi_{self} = 183.79$, which was lower than the average reference profit of its neighbours $\pi_{nbs} = 189.28$ and therefore the firm dropped its price to 8.

We will now describe one particular series of random events that leads firm (2,2) to adapt price 7, which is a price not yet present in (this part of) the population. First, we let the random mechanism select firm (2,3) as the subject. The cheapest neighbour sets a price of 9, the subject realizes a profit of $\pi_{self} = 195.20$ and $\pi_{nbs} = 189.28$. The subject is in a 'win' situation and therefore it updates its price to 9. This situation is depicted in Figure 4, which is the trigger point.

6 For convenience, the figures only present rounded values of the profits. The arguments are based upon the unrounded values.

7 Note that this is only a small part of the full 11×11 payoff table.

9 189	9 189	9 189	9 189
9 189	9 189	9 195	9 189
9 189	9 189	9 189	9 189

Figure 4 – The 3×4 subsection: The trigger point

Next, let firm (2,2) be selected as the subject. Its cheapest neighbour sets price 9, and the subject realizes a profit equal to $\pi_{self} = 189.91 < \pi_{nbs}$, because $\pi_{\text{nbs}} = \frac{7}{8} \cdot 189.28 + \frac{1}{8} \cdot 195.20 = 190.02$. Hence, the subject is in a 'lose' situation and subsequently will change its price to 8 leading to the situation in Figure 5.

9 189	9 189	9 189	9 189
9 189	8 190	9 195	9 189
9 189	9 189	9 189	9 189

Figure 5 – The 3×4 subsection: Firm (2,2) adjusts to price 8

Now suppose the random mechanism selects firm (1,3) as the subject, although any of the neighbours of firm (2,2), except firm (2,3), would do. This

9 189	9 189	8 184	9 189
9 189	8 190	9 195	9 189
9 189	9 189	9 189	9 189

Figure 6 – The 3×4 subsection: Firm (1,3) adjusts to price 8

subject will compete against the cheapest competitor which sets price 8 (not necessarily firm (2,2)) and will obtain a profit of 183.79. We see the subject is in a 'lose' situation and the firm lowers its price to 8. We have now arrived in the situation of Figure 6.

As a last event, let the random mechanism once more select firm (2,2) as the subject. Its cheapest neighbour is (1,3) which sets price 8. Therefore, firm (2,2) will get a profit of $\pi_{self} = 189.28 < \pi_{nbs}$, where $\pi_{nbs} = \frac{6}{8} \cdot 189.28 + \frac{1}{8} \cdot 195.20 + \frac{1}{8} \cdot 183.79 = 189.33$. This again puts the subject in a 'lose' situation and therefore it lowers its price to 7 as depicted in Figure 7.

9 189	9 189	8 184	9 189
9 189	7 189	9 195	9 189
9 189	9 189	9 189	9 189

Figure 7 – The 3×4 subsection: Start of a price war

At this point, i.e. Figure 7, the population is no longer in a stable situation. Instead it has entered one of the small time intervals $[t_1, t_2]$, were the local price war will spread out to the neighbours. Since there is now one firm that sets a price 7, other firms which compete with this firm will be in a 'lose' situation and will therefore also lower their prices. The latter scenario is eight times more likely than the event that firm (2,2) will adjust upwards to price 8 (i.e. the local price war does not spread). Therefore, it is most likely that the population as a whole will move in the direction away from the quasi-stable state $\bar{s}$. After some stochastic time however, there will be enough firms with sufficiently low profits the last time they were the subject to ensure that some firms as the subject will realize a profit higher than the average profit of their neighbours and thus will be in 'win' situation again. Then the trend away from the state $\bar{s}$ is reversed and there is again convergence towards the same quasi-stable state $\bar{s}$ as before. The essential feature of the trigger point is that one firm is selected twice in very short succession, i.e. before any of its neighbours has been selected more than once. The events as described above have a very small probability of happening, but since there are many situations alike in which a trigger point is reached, the overall probability of reaching a trigger point is large enough to observe the effects very clearly. Moreover, since competition is always between the subject and its most competitive neighbour, having just one firm that sets a low price in the neighbourhood of the subject is enough to get the subject to lower its price. This way, the specified interaction structure contributes to a large extent to the possibility for the low price to spread out rapidly over (a part of) the population.

5 CONCLUDING REMARKS

In this paper we reported that collusive behaviour can evolve in a social environment consisting of horizontally differentiated firms which compete locally in prices and which follow the explicit behavioural rule WCLD. Comparative statics reveal that our results are robust. Furthermore, the average percentage of collusive behaviour increases the finer the grid of discrete prices is, which is due to a weaker price adjustment when the grid is finer. Price dispersion and occasional price wars coexists.

In section 4 we have argued that a specific chain of events of the random process which selects the subject induces one of the firms to lower its price below any price that is played in its neighbourhood. This can be viewed as a spontaneous shock to the dynamics of the model (a so-called mutation in the terminology of evolutionary game theory), which arises endogenously in the model. This differs with the standard evolutionary models by Young (1993), Young and Foster (1991), Kandori, Mailath, and Rob (1993), and Ellison (1993, 2000) where spontaneous mutations are exogenous features that change the outcome of the models in an essential way.

Finally, in our model, a price war is an endogenous feature which starts at unpredictable moments in time. These wars depend upon a change in behaviour of only a few firms under specific circumstances, which we have clarified. The time at which a price war starts and the time it lasts are both stochastic. Put in a broader perspective, our results can be seen as an alternative explanation for the bubble-crash cycles, as observed by e.g. Smith, Suchanek, and Williams (1988), suggesting that it might not be mere risk aversion that generates the cycles observed by these authors. The feature of price wars warrants further detailed mathematical study in future research.

APPENDIX

A QUASI-STABLE STATES

In this appendix several quasi-stable states are reported for $k = 9,19,29$. The top row of each table represents the possible prices a, $a = 0, \ldots, k$, which correspond to the prices $p^N + \frac{a}{k}(p^C - p^N)$. The bottom row reports the percentage of the firms that set the price a. The average percentage of collusion is reported in the last column, which is simply the average of the prices weighted by the percentage of the firms playing that price.

$k = 9$:

Price a	0	1	2	3	4	5	6	7	8	9	Av.% a
% of Firms	0.0	0.0	0.0	0.0	0.0	0.2	7.6	28.6	39.7	23.9	86.6

$k = 19$:

Price a	0	1	2	3	4	5	6	7	8	9	10
% of Firms	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Price a	11	12	13	14	15	16	17	18	19	Av.% a
% of Firms	0.0	0.0	0.0	0.0	0.8	9.1	28.6	38.3	23.2	93.4

$k = 29$:

Price a	0	1	2	3	4	5	6	7	8	9	10
% of Firms	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Price a	11	12	13	14	15	16	17	18	19	20	21
% of Firms	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Price a	22	23	24	25	26	27	28	29	Av.% a
% of Firms	0.0	0.0	0.0	0.6	9.0	28.2	38.2	24.0	95.7

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